Assignment 1

GCM 2010

Instructions. Answer the following, and be prepared to present and discuss your answers in class.

1. Suppose X has a mean of 10. What is the mean of Y if $Y = 2X - 4$?

2. Suppose X has a mean of 140 and a standard deviation of 20. What linear transformation of X will create a new set of scores Y with a mean of 100 and a standard deviation of 15?

3. Suppose that you have a random variable ξ representing the "true score" of a population, and that ξ is normally distributed with a mean of zero variance of 1. Consider two tests X_1 and X_2 that impose random, normally distributed error on ξ. Assume that the random error random variables ϵ_1 and ϵ_2 have means of zero and variances of 1, and both are uncorrelated with each other and ξ. Imagine that $X_1 = \rho^{1/2}\xi + (1-\rho)^{1/2}\epsilon_1$, and $X_2 = \rho^{1/2}\xi + (1-\rho)^{1/2}\epsilon_2$.

   Prove the following, using well known results on the algebra of linear transformations and combinations:

   (a) X_1 and X_2 have means of zero.
   (b) X_1 and X_2 have a covariance of ρ
   (c) X_1 and X_2 have a correlation of ρ
   (d) X_1 and X_2 have variances of 1.
   (e) X_2 has a covariance of $\rho^{1/2}$ with ξ

4. Using the same notation as the previous problem, we can say that $X_1 = T + E_1$, where $T = \rho^{1/2}\xi$, and $E_1 = (1-\rho)^{1/2}\epsilon_1$. What percentage of the variance of X_1 is accounted for by the variance of T? This value is known as the *reliability* of X_1 in classical test theory.

5. Consider the following data: $x = [1, 2, 3]$, $y = [1, 4.2, 8.9]$.

   (a) Using R, plot these data as 3 red points in the plane, and then see if you can find at least 2 functions that (a) pass close to the points, but (b) are rather different in some important sense over the range $0 \leq x \leq 10$. Plot these two functions in the plane, in different colors, along with the points. Make sure to include your R code.

(b) What is a key lesson of the previous exercise for growth curve analysis?